**# 1D Harmonic Oscillator**

Please make code to plot wave function solved by schroedinger equation. This problem is harmonic oscillator.

can you expand range of the x-axis from 10 to 100 and draw wave functions from ground state to 9th excited state?

please draw wave functions seperately.

코드의 핵심.

E, psi = np.linalg.eigh(T + V\_mat)

numpy library에서 hermitian operator가 주어지면, numerical하게 hermitian operator의 eigen value와 eigen function 구하는 함수.

**# 정성적 해석에 대한 이해도를 알기 위한 질문**

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can you explain why the nodes increase when n\_states increase?

Yes, when we increase the number of states (n\_states) that we plot, we are plotting the wave functions corresponding to higher and higher energy levels. In the harmonic oscillator, the energy levels are quantized, meaning that only certain energies are allowed. These energies increase as we move to higher energy levels.

The wave functions corresponding to different energy levels have different numbers of nodes. A node is a point where the wave function passes through zero. The ground state (the lowest energy state) has no nodes, meaning that the wave function is always positive or always negative. The first excited state (the second lowest energy state) has one node, meaning that the wave function changes sign once. The second excited state (the third lowest energy state) has two nodes, meaning that the wave function changes sign twice, and so on.

Therefore, as we increase the number of states we plot, we are plotting wave functions corresponding to higher and higher energy levels, which have more and more nodes. This is why the number of nodes increases when n\_statesis increased.

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can you explain number of node and wavelength of the eigen function?

Certainly! In quantum mechanics, the wave function (also known as the eigenfunction) describes the probability amplitude of a particle's position or other observable quantities. The wave function can have nodes, which are points where the wave function passes through zero. The number of nodes in the wave function is related to its wavelength, which is the distance between two consecutive nodes.

The wavelength of the wave function is related to the momentum of the particle. In quantum mechanics, particles can behave as waves, and the wavelength of the wave function is inversely proportional to the momentum of the particle:

λ = h/p

where λ is the wavelength, h is Planck's constant, and p is the momentum of the particle. Therefore, the wavelength of the wave function increases as the momentum of the particle decreases.

The number of nodes in the wave function is related to the energy of the particle. In the case of the harmonic oscillator, the energy of the particle is quantized, meaning that only certain energies are allowed. The energy of the particle increases as the number of nodes in the wave function increases. The ground state (the lowest energy state) has no nodes, while the first excited state (the second lowest energy state) has one node, the second excited state (the third lowest energy state) has two nodes, and so on.

In summary, the number of nodes in the wave function is related to the energy of the particle, while the wavelength of the wave function is related to the momentum of the particle. As the energy of the particle increases, the number of nodes in the wave function also increases, while the wavelength of the wave function decreases. Conversely, as the momentum of the particle increases, the wavelength of the wave function decreases, while the number of nodes in the wave function remains the same.

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can you calculate wavelength of each eigen wave function when eigen energy increases? Please explain the trend qualitatively.

Sure, we can calculate the wavelength of each eigen wave function using the formula:

λ = 2π/k

where λ is the wavelength and k is the wave number. The wave number is related to the energy of the particle through the formula:

E = (hbar^2 k^2)/(2m)

where E is the energy, hbar is the reduced Planck's constant, and m is the mass of the particle. Therefore, we can write:

k = sqrt(2mE)/hbar

Substituting this expression for k into the equation for the wavelength, we get:

λ = 2π(hbar/sqrt(2mE))

The wavelength is therefore inversely proportional to the square root of the energy.

In the case of the harmonic oscillator, the energy levels are quantized, meaning that only certain energies are allowed. As we move to higher energy levels, the energy increases, and the wavelength of the corresponding eigen wave functions decreases. This means that the wave functions become more and more tightly packed, with shorter and shorter wavelengths.

Qualitatively, this trend makes sense because as the energy of the particle increases, its momentum also increases, as we saw in my previous response. This means that the particle moves faster and has a shorter wavelength. Additionally, as the energy of the particle increases, its amplitude becomes more spread out in space, leading to a more rapidly varying wave function with shorter wavelength. Therefore, we expect the wavelength of the eigen wave functions to decrease as the energy increases, leading to more and more tightly packed wave functions with shorter and shorter wavelengths.

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Can you explain the phenomenon that the amplitude of wave functions become higher when x increases? Especially, the amplitude of wave function is highest at the edge of the wave function.

Sure! The phenomenon you are referring to is related to the concept of probability density in quantum mechanics. The absolute square of the wave function gives us the probability density of finding the particle at a particular position. In other words, the amplitude of the wave function at a particular point is related to the probability of finding the particle at that point.

In the case of the harmonic oscillator, the ground state wave function has a Gaussian shape centered around the origin, with the highest probability density at x=0. As we move to higher energy levels, the wave functions become more spread out, and their probability density is spread over a larger range of x.

Now, let's consider why the amplitude of the wave function becomes higher as x increases, especially at the edges of the wave function. In quantum mechanics, the wave function is a complex-valued function that describes the probability amplitude of finding the particle at a particular position. The amplitude of the wave function at a particular point is determined by the interference of different waves that make up the wave function. When these waves interfere constructively, the amplitude of the wave function is high, and when they interfere destructively, the amplitude is low.

In the case of the harmonic oscillator, the wave functions are symmetric about the center, which means that the wave function at positive x is the mirror image of the wave function at negative x. This symmetry leads to constructive interference at the edges of the wave function, where the two halves of the wave function interfere constructively, resulting in a higher amplitude. In contrast, the amplitude of the wave function is low at the center, where the two halves of the wave function interfere destructively.

Therefore, we see that the amplitude of the wave function becomes higher as x increases, especially at the edges of the wave function, due to the constructive interference of the two halves of the wave function.

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can you explain the trend of momentum when x changes?

In quantum mechanics, the momentum of a particle is related to the wave function through the Fourier transform. The wave function and its Fourier transform are related by the following equation:

ψ(p) = (1/√(2πħ)) ∫ exp(-ipx/ħ) ψ(x) dx

where ψ(x) is the wave function, ψ(p) is the momentum wave function, p is the momentum, x is the position, ħ is the reduced Planck's constant, and the integral is taken over all values of x.

This equation tells us that the momentum wave function is the Fourier transform of the position wave function. The momentum wave function gives us the probability amplitude of finding the particle with a particular momentum.

Now, let's consider the trend of momentum when the position changes. The momentum wave function for the harmonic oscillator is a Gaussian function centered around zero momentum. This means that the most probable momentum of the particle is zero, and the probability of finding the particle with a non-zero momentum decreases as the momentum increases in magnitude.

As the position of the particle changes, the shape of the wave function changes, and its Fourier transform (the momentum wave function) also changes. Specifically, the peak of the momentum wave function becomes narrower as the position of the particle moves away from the center of the potential well. This means that the particle has a more well-defined momentum when it is farther away from the center of the potential well. Additionally, the momentum wave function becomes more spread out as the position of the particle moves closer to the center of the potential well. This means that the particle has a less well-defined momentum when it is closer to the center of the potential well.

Therefore, we see that as the position of the particle changes, the momentum wave function changes, and the momentum of the particle becomes more or less well-defined depending on the position of the particle relative to the center of the potential well.

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can you write the exact wave function in the momentum space from ground state to 9th excited state?

Sure! The wave function in the momentum space (momentum eigenfunction) for the harmonic oscillator is given by:

ψ\_n(p) = (1/√(2^n n! √(π) ħ)) H\_n(p/√(ħmω)) exp(-p^2/(2ħmω))

where n is the quantum number, ħ is the reduced Planck's constant, m is the mass of the particle, ω is the frequency of the oscillator, H\_n(x) is the nth Hermite polynomial, and p is the momentum.

For the ground state (n=0), the momentum wave function is given by:

ψ\_0(p) = (1/√(π ħmω)) exp(-p^2/(2ħmω))

For the first excited state (n=1), the momentum wave function is given by:

ψ\_1(p) = (1/√(2 π ħmω)) 2^(1/4) p exp(-p^2/(2ħmω))

For the second excited state (n=2), the momentum wave function is given by:

ψ\_2(p) = (1/√(8 π ħmω)) (p^2-ħmω) exp(-p^2/(2ħmω))

And so on, for higher energy states.

Note that the momentum wave function for the harmonic oscillator is always a Gaussian function centered at zero momentum, which means that the most probable momentum of the particle is zero. However, as the energy of the state increases, the width of the Gaussian function decreases, which means that the momentum becomes more well-defined.

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But you don't need to use the wave function in momentum space to explain the trend of momentum. Please think kinetic energy when x changes, and then explain the trend of momentum using the change of kinetic energy when x changes.

You're correct, I apologize for the confusion in my previous response. Let me try to provide a more accurate explanation.

As the position of a particle in the harmonic oscillator changes, its kinetic energy changes. The kinetic energy of the particle is given by:

K = (p^2)/(2m)

where p is the momentum of the particle and m is its mass.

When the particle is near the center of the oscillator, it has a small potential energy and a large kinetic energy. As it moves away from the center, the potential energy increases and the kinetic energy decreases. At the turning points, where the particle's velocity is zero, its kinetic energy is entirely converted into potential energy.

The momentum of the particle is related to its kinetic energy by the following equation:

p = √(2mK)

This equation tells us that the momentum of the particle increases as its kinetic energy increases.

When the particle is near the center of the oscillator, it has a large kinetic energy and therefore a large momentum. As it moves away from the center, its kinetic energy decreases, and its momentum decreases as well. At the turning points, the kinetic energy is zero, and the momentum is also zero.

Therefore, we see that the momentum of the particle changes as its kinetic energy changes, and this change in momentum is related to the change in the particle's position relative to the center of the potential well. When the particle is closer to the center of the oscillator, it has a higher kinetic energy and a higher momentum, and as it moves away from the center, its kinetic energy and momentum decrease.

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then, can you explain the correlation of momentum of particle and amplitude of the wave function in position space?

Yes, there is a correlation between the momentum of a particle and the amplitude of its wave function in position space.

The amplitude of the wave function in position space gives us information about the probability of finding the particle at a given position. The momentum of the particle, on the other hand, is related to the wavelength of its wave function in position space.

The momentum of a particle can be calculated from its wave function in position space by taking the Fourier transform of the wave function. The Fourier transform converts a function from position space to momentum space, and vice versa. The relationship between the position and momentum wave functions is given by the uncertainty principle, which states that the product of the uncertainty in position and the uncertainty in momentum must be greater than or equal to a certain constant:

Δx Δp ≥ ħ/2

where Δx is the uncertainty in position, Δp is the uncertainty in momentum, and ħ is the reduced Planck constant.

As a result of the uncertainty principle, if the position of a particle is well-defined, meaning that Δx is small, then its momentum must be less well-defined, meaning that Δp is large. Conversely, if the momentum of a particle is well-defined, meaning that Δp is small, then its position must be less well-defined, meaning that Δx is large.

So, in summary, the amplitude of the wave function in position space gives us information about the probability of finding the particle at a given position, while the wavelength of the wave function is related to the momentum of the particle. When the amplitude of the wave function is high at a particular position, it means that the probability of finding the particle at that position is high. At the same time, the wavelength of the wave function at that position is inversely proportional to the momentum of the particle. Therefore, when the amplitude of the wave function is high at a particular position, the momentum of the particle at that position is likely to be low, and vice versa.

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You're right and thanks for your interpretation using uncertainty. Then, please check whether the next sentence is true or false. "Probability to find the particle in the position increases when the momentum of the particle decreases."

This statement is not necessarily true. The probability to find the particle in a particular position is determined by the amplitude of its wave function in position space at that position. The momentum of the particle is related to the wavelength of its wave function in position space, and the relationship between the amplitude and wavelength of the wave function is governed by the uncertainty principle.

Therefore, the probability of finding the particle in a particular position is not directly related to the momentum of the particle. It is possible for the amplitude of the wave function to be high at a particular position, indicating a high probability of finding the particle at that position, while the momentum of the particle at that position is also high. Similarly, it is possible for the amplitude of the wave function to be low at a particular position, indicating a low probability of finding the particle at that position, while the momentum of the particle at that position is also low.

In summary, the probability of finding the particle in a particular position is determined by the amplitude of its wave function in position space, while the momentum of the particle is related to the wavelength of its wave function. These two quantities are related by the uncertainty principle, but the probability of finding the particle is not directly determined by its momentum.

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Oh I see. In classical Harmonic Oscillator, is the probability to find the particle at the position increases when the momentum of particle decreases?

In classical mechanics, the concept of probability density does not exist. The position and momentum of a particle in classical mechanics are described by exact, deterministic values at any given time, and there is no inherent uncertainty in the position or momentum of the particle.

Therefore, in classical mechanics, there is no question of the probability to find the particle at a particular position. The position and momentum of the particle are precisely determined by the initial conditions of the system and the equations of motion that govern the dynamics of the system.

In the case of the classical harmonic oscillator, the position and momentum of the particle are related by the equation:

p = mωx

where p is the momentum of the particle, x is its position, m is its mass, and ω is the frequency of the oscillator. This equation shows that the momentum of the particle is directly proportional to its position in the oscillator, and the proportionality constant is mω.

Therefore, in classical mechanics, the position and momentum of the particle in the harmonic oscillator are not independent quantities, and there is no question of the probability to find the particle at a particular position. Instead, the position and momentum of the particle are related by a precise mathematical relationship, and the dynamics of the system are fully determined by the initial conditions and the equations of motion.

**# 2D harmonic oscillator**

Please make code to plot wave function solved by schroedinger equation. This problem is harmonic oscillator in 2 dimension.

at the code ax.plot\_surface(X, Y, prob\_density, cmap='plasma'), there is error. next is error message.

at the code alpha = np.sqrt(hbar / (m\*w)), there is error message : float division by zero. please correct this error.

I think there is duplicated m : particle mass and quantum number. please use another letter for particle mass.

probability density is plotted zero for all position. please correct this problem.

please plot probability density of each eigen states seperately.

1D Harmonic Oscillator보다는 학습이 덜 되어서 그런지... 문법적 오류 발생 빈도 증가

에러 메시지 주고 에러 수정하라고 지시.

particle mass m과 eigen state number m 중복 사용.

**# 2D infinite well**

then, please make code to plot wave function solved by schroedinger equation when the potential is 2 dimensional infinite well.

please plot time independent wave function.

there is an error message : shape mismatch: objects cannot be broadcast to a single shape. Mismatch is between arg 0 with shape (100, 100) and arg 2 with shape (10000, 10000).

I think there is duplicated alphabet m, particle mass and eigen state. please use another letter to represent the paritcle mass. and please plot when eigen state is (1,1), (1,2), (2,1), (2,2), (3,1), (1,3), (3,2), (2,3), (3,3).

there's error message : shape mismatch: objects cannot be broadcast to a single shape. Mismatch is between arg 0 with shape (100, 100) and arg 2 with shape (10000, 10000). correct your code please.

역시 문법적 오류와 m 중복 문제.

**# Pure Quartic Potential**

chat gpt에 수십번 질문해서 겨우 어느정도 비슷한 wave function과 eigen energy 그래프 얻는 데 성공.

1. simple harmonic oscillaor와 혼동해서 potential energy에 quadratic function을 넣는 문제

2. simple harmonic oscillator에서의 pertubation 문제와 혼동하여 pure quartic potential이 아니라 symmetrical double well이나 simple harmonic oscillator에서 람다가 작을 때 pertubation 문제에서 도출된 eigen energy를 pure quartic potential의 energy로 사용하는 문제 발생.

3. 그 외 다수의 문법적 오류 발생.

문제를 해결하기 위해... chat gpt에게 numpy.linalg.eigh 사용하도록 지시.

do not use given energy. use numpy.linalg.eigh to gain eigen energy for pure quartic potential. please make code to plot each wave functions seperately.